Chapter 3

1

Motion in one dimension

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1

Lecture 02

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Acceleration and Kinematic Equations

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2

Average Acceleration

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Acceleration is the rate of change of the velocity.

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

SI units are m/s^2

Vector quantity

In one dimension, positive and negative can be used to indicate direction.

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Average Acceleration

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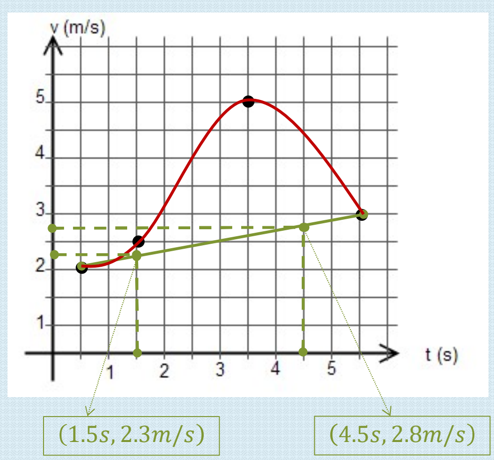
Velocity – time graph

$$slope\ of\ the\ straight\ line = a_{avg}$$

$$= \frac{\Delta v}{\Delta t} = \frac{2.8 - 2.3}{4.5 - 1.5}$$

$$= 0.17\ m/s^2$$



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Instantaneous Acceleration

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- It is the acceleration at a certain time.
- It is the slope of the line tangent to the *velocity – time* curve.
- The first derivative of the velocity with respect to time:

$$\boldsymbol{a} = \frac{dv}{dt}$$

- The second derivative of the position with respect to time:

$$\boldsymbol{a} = \frac{d^2x}{dt^2}$$

- The instantaneous acceleration can be positive, negative, or zero.

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Instantaneous Acceleration

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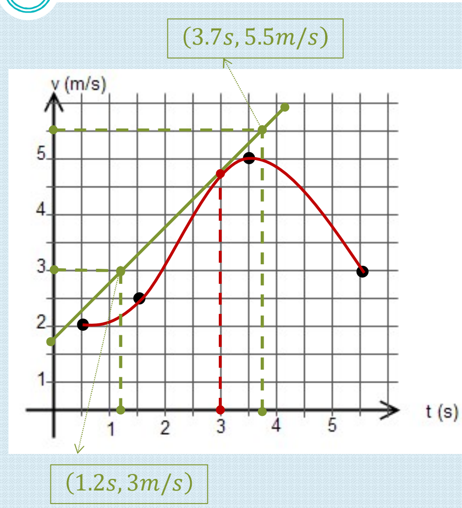
Velocity – time graph

$$slope\ of\ the\ tangent = a\Big|_{t=3s}$$

$$= \frac{\Delta v}{\Delta t} = \frac{5.5-3}{3.7-1.2}$$

$$= \frac{2.5}{2} = 1.25\ m/s^2$$



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6

Integrating velocity ≡ Displacement

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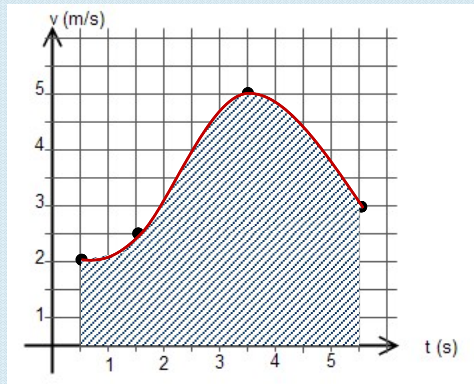
Velocity – time graph

$$v = \frac{dx}{dt}$$

$$\Delta x = \int_{t_i}^{t_f} v\, dt$$

$$\Delta x = Area\ under\ v - t\ graph$$



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7

Integrating acceleration ≡ Change in velocity

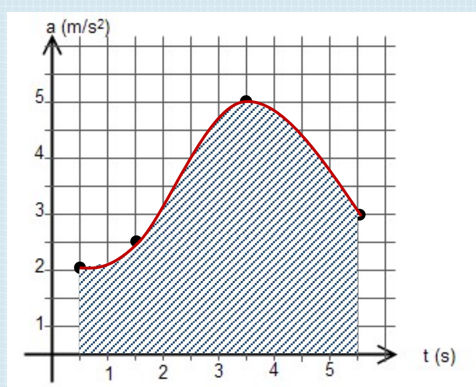
8

acceleration – time graph

$$a = \frac{dv}{dt}$$

$$\Delta v = \int_{t_i}^{t_f} a\, dt$$

$$\Delta v = Area\ under\ a - t\ graph$$



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8

Acceleration and Velocity, Directions

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- When an object's velocity and acceleration are in the same direction, the object is speeding up.
- When an object's velocity and acceleration are in opposite directions, the object is slowing down.
- When the object's velocity is constant, the acceleration is zero.
- When the object's acceleration is constant, NOTHING to conclude.
- Force and acceleration are both vectors and directed in the same direction.
- The word deceleration has the connotation of slowing down.

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Notes About Acceleration

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- Negative acceleration does not necessarily mean the object is slowing down.
  - If the acceleration and velocity are both positive, the object is speeding up.
  - If the acceleration and velocity are both negative, the object is speeding up.
  - If the acceleration and velocity are of opposite signs, the object is slowing down.

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Producing An Acceleration

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Various changes in a particle's motion may produce an acceleration.

- The magnitude of the velocity vector may change.
- The direction of the velocity vector may change.
- Both may change simultaneously

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Constant Velocity

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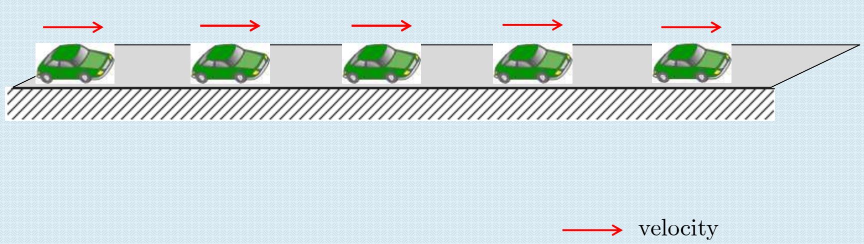
- Images are equally spaced.
- The car is moving with constant positive velocity (shown by red arrows maintaining the same size).
- Acceleration equals zero.



12

Acceleration and Velocity

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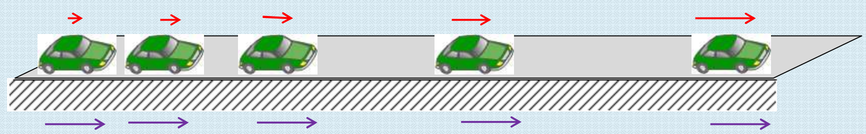
- Images become farther apart as time increases.
- Velocity and acceleration are in the same direction.
- Acceleration is uniform (violet arrows maintain the same length).
- Velocity is increasing (red arrows are getting longer).
- This shows positive acceleration and positive velocity.



acceleration velocity

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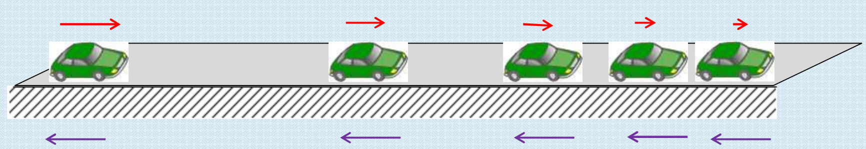
Acceleration and Velocity

14

- Images become closer together as time increases.
- Acceleration and velocity are in opposite directions.
- Acceleration is uniform (violet arrows maintain the same length).
- Velocity is decreasing (red arrows are getting shorter).
- Positive velocity and negative acceleration.



acceleration velocity

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A Particle Under Constant Acceleration

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Constant acceleration indicates that at any instant during a time interval the instantaneous acceleration is the same as the average acceleration.

$$a = a_{avg}$$

$$a = \frac{v_f - v_i}{t_f - t_i}$$

Common practice is to let $t_i = 0$, $t_f = t$ and the equation becomes:

$$v_f = v_i + at$$

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Kinematic Equations

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- The kinematic equations can be used with any particle under uniform acceleration.
- The kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration.
- You may need to use two of the equations to solve one problem.
- Many times there is more than one way to solve a problem.

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Kinematic Equations

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Equations	Missing
$v_f = v_i + at$	Δx : displacement (m)
$v_f^2 = v_i^2 + 2a\Delta x$	t : time (s)
$\Delta x = v_i t + \frac{1}{2}at^2$	v_f : final velocity (m/s)
$\Delta x = v_f t - \frac{1}{2}at^2$	v_i : initial velocity (m/s)
$\Delta x = \frac{1}{2}(v_i + v_f)t$	a : acceleration (m/s²)

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When a = 0

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When the acceleration is zero,

$$v_f = v_i = v$$

$$x_f = x_i + vt$$

The constant acceleration model reduces to the constant velocity model.

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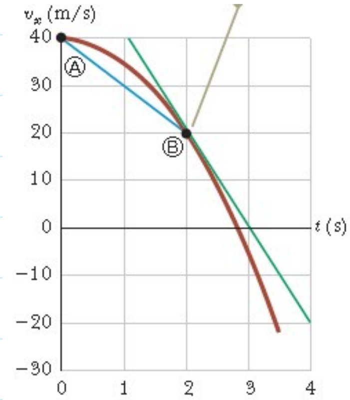
Average and Instantaneous Acceleration-1

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.



The velocity of a particle moving along the x axis varies according to the expression: $v = 40 - 5t^2$, where (v) is in (m/s) and (t) is in seconds.

- Find the average acceleration in the time interval (t = 0) to (t = 2 s).
- Determine the acceleration at (t = 2 s).

(A) Solution

Find the velocities at ti = tA = 0 and tf = t_B = 2.0 s by substituting these values of t into the expression for the velocity:

$$v_{x_A} = 40 - 5t_A^2 = 40 - 5(0)^2 = +40m/s$$

$$v_{x_B} = 40 - 5t_B^2 = 40 - 5(2.0)^2 = +20m/s$$

Find the average acceleration in the specified time interval

$$a_{x,avg} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{x_B} - v_{xA}}{t_B - t_A} = \frac{20m/s - 40m/s}{2.0s - 0s} = -10m/s^2$$

(B) Solution

Differentiating the position function, we find:

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(40 - 5t^2) = -10t$$

Find the acceleration at t=2.0 s

$$a_x\Big|_{t=2s} = (-10)(2.0)m/s^2 = -20m/s^2$$

Because the velocity of the particle is positive and the acceleration is negative at this instant, the particle is slowing down.

Average and Instantaneous Acceleration-2

Friday, 10 September, 2021 09:09

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [ ] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [x] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A particle's position on the x axis of is given by $x = t^3 - 27t + 4$ with x in meters and t in seconds. When the particle is at rest, what is its acceleration?

Solution

Differentiating the position function, we find:

$$v(t) = -27 \ + 3t^2$$

Differentiating the velocity function then gives us:

$$a(t) = +6t$$

Setting v(t) = 0 yields

$$0 = -27 \ + \ 3t^2 \ \Rightarrow \ t = \ \mp 3 \text{ s.}$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

$$a(t) = +6t \ \Rightarrow \ a(t = 3) = 6 \times 3 = 18 m/s^2$$

At $t = 3s$, $v = 0 m/s$ and $a = 18 m/s^2$. Then, the particle is about to move.

Carrier Landing

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A jet lands on an aircraft carrier at a speed of 63 m/s.

- What is its acceleration if it stops in 2 s?
- What is its final position?

(A) Solution

We use the equation that does not involve position:

$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{0 - 63}{2} = -31.5 m/s^2$$

(B) Solution

We use the equation that does not involve acceleration:

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t = 0 + \frac{1}{2}(63 + 0)(2) = 63m$$

Uniform acceleration

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A car accelerates uniformly from rest to a speed of $100\ km/h$ in $18\ s$.

- Find the acceleration of the car.
- Find the distance that the car travels.
- If the car brakes to a full stop over a distance of $100\ m$, then find its uniform deceleration.

Determine the corresponding speed in units of (m/s)

$$v = 100\ \frac{km}{hr} = 100\ \frac{km}{hr} \times \frac{1000\ m}{1\ km} \times \frac{1\ hr}{60\ min.} \times \frac{1\ min.}{60\ s}$$

$$= \frac{100 \times 1000}{60 \times 60}\ \frac{m}{s} = 27.8\ m/s$$

(A) Solution

We use the equation that does not involve position:

$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{27.8 - 0}{18} = 1.54\ m/s^2$$

(B) Solution

We use the equation that does not involve acceleration:

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t = 0 + \frac{1}{2}(0 + 27.8)(18) = 250.2\ m$$

(B) Solution

We use the equation that does not involve time:

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{0^2 - 27.8^2}{2 \times 100} = -3.86\ m/s^2$$

Quadratic equation

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A remote-controlled truck moves along the x-axis with a constant acceleration of $-2\ m/s^2$. As it passes the origin, its initial velocity is $14\ m/s$.

(a) At what time does $v = 0$ (i.e. when the truck stops momentarily)?

(b) At what position does $v = 0$ (i.e. when the truck stops momentarily)?

(c) At what time is the truck at $x = 24\ m$? and

(d) What is its velocity then?

(a) Solution:

We use the equation that does not involve position:

$$t = \frac{v_f - v_i}{a} = \frac{0 - 14}{-2} = 7s$$

(b) Solution:

We use the equation that does not involve time:

$$x_f = x_i + \frac{v_f^2 - v_i^2}{2\mathrm{a}} = 0 + \frac{0^2 - 14^2}{2 \times (-2)} = 49\ m$$

(c) Solution:

We use the equation that does not involve final velocity:

$$\mathrm{x_f} = x_i + v_i t + \frac{1}{2} a t^2$$

$$24 = 0 + 14t + \frac{1}{2} \times (-2) t^2$$

$$t^2 - 14t + 24 = 0$$

Solving this quadratic equation yields:

$$t = \{2s, 12s\}$$

Thus, $t_1 = 2\ s$ is the time the truck takes from the origin to the position $x = 24m$.

Furthermore, $t_2 = 12\ s$ is the time the truck takes from the origin, passing the point $x = 24m$, reaching the point $x = 49\ m$ and returning back to $x = 24m$.

(d) Solution:

At $t_1 = 2\,s$, we use the equation that does not involve position:

$$v_f = v_i + at = 14 + (-2 \times 2) = 10\,m/s$$

At $t_2 = 12\,s$, we use the equation that does not involve position:

$$v_f = v_i + at = 14 + (-2 \times 12) = -10\,m/s$$

Observe that both velocities are equal in magnitude, but in opposite directions.